# Indira Gandhi University, Meerpur (Rewari)



Scheme of Examination

for

M.Sc.(Mathematics)

w.e.f. session 2019-20

as per

Choice Based Credit System (CBCS)

# Indira Gandhi University, Meerpur (Rewari) Scheme of Examination M.Sc. Mathematics Under Choice Based Credit System w.e.f. Session 2019-20

# Semester-I

# Core Courses

Course	Title of the Course	Theory	Internal	Practical	Credits	Contact	Total
Code		Marks	Marks	Marks	L:T:P	hrs	Credits
						per week	
MAT-101	Abstract Algebra	80	20	-	4:0:0	4	4
MAT-102	Mathematical Analysis	80	20	-	4:0:0	4	4
MAT-103	Ordinary Differential	80	20	-	4:0:0	4	4
	Equations						
MAT-104	Complex Analysis	80	20	-	4:0:0	4	4
MAT-105	Mathematical Statistics	80	20	-	4:0:0	4	4
MAT-106	Computer Applications	60	-	40	2:0:2	6	4
MAT-107	Mathematical Lab-I	-	-	50	0:0:2	4	2
MAT-108	Seminar	-	-	25	-	-	1
MAT-109	Self Study Paper	-	-	25	-	-	1

### Total Credits : 28 Total Contact Hours per Week : 30 Max Marks : 700

Note: The criteria for awarding internal assessment of 20 marks for each paper shall be as under :

(i) Sessional test	: 10 marks
(ii) Assignment/Presentation	: 5 marks
(iii) Attendance	: 5  marks
Less than $65\%$	: 0 marks
65% and above but upto $70%$	: 2  marks
Above 70% but up to 75%	: 3  marks
Above 75% but up to 80%	: 4 marks
Above 80%	: 5  marks

# <u>General Guidelines</u>

### 1. Seminar

In each semester, there will be a paper on seminar presentation of 25 marks with 01 credit. In this paper, the student will be required to present a seminar of about 15-20 minutes on the theme/topic such as review of research papers/articles published in National/International Journals in his /her area of interest. The topic will be selected by the student in consultation with the teacher alloted to him/her by the department.

An internal committee of two teachers constituted by the Chairperson of the department for each student will evaluate the seminar presentation. The evaluation (Internal evaluation only) will be based on the presentation of student, depth of subject matter and answer to questions. There will be a Coordinator to be nominated by the Chairperson of the Department among the teachers of the Department.

1st Semester	Any topic (not related to the syllabi)
2nd Semester	Any Basic Research Paper/Article
3rd Semester	Any National Level Research Paper/Article
4th Semester	Any Foreign Research Paper/Article

For seminar, the topics should be chosen in the following manner:

#### 2. Self Study Paper

In each semester, there will be a self study paper of 25 marks with 01 credit. The objective of this paper is to create habits of reading books and to develop writing skills in a manner of creativity and originality. The students will select a topic of their own interest in the given area in consulation with their teachers/incharge/mentors. After selecting a suitable title for the paper, the students will be required to prepare a hand written report of about 6-10 pages in his/her own handwritng. The students will be required to submit the report after getting it checked by the concerned teacher and will be asked to re-submit the report after making the required corrections(if any) before the commencement of the examinations of that semester. The structure of the paper will include the following:

- Introduction
- Main Body
- Conclusion

The thoughts presented in the paper must be original work of the students.

The paper will be evaluated by the panel (one external and one internal examiner) to be appointed by the Chairperson of Department from the prescribed panel of the University.

The evaluation of Self Study paper will be done as given below:

- Evaluation of the paper : 15 marks
- Viva-Voce on the paper : 10 marks
- Total : 25 marks

# MAT-101: Abstract Algebra

### Time : 3 hours

Max. Marks : 80 Credits : 4:0:0

Note: The question paper will consist of five Sections. Each of the sections I to IV will contain two questions and the students shall be asked to attempt one question from each. Section-V shall be compulsory and will contain eight short answer type questions without any internal choice covering the entire syllabus.

#### Section-I

*p*-groups, Sylow *p*-subgroups, Sylow theorems, Applications of Sylow theorems, Description of groups of order  $p^2$  and pq, Survey of groups upto order 15.

### Section-II

Normal and subnormal series, Solvable series, Derived series, Solvable groups, Solvability of  $S_n$ -the symmetric group of degree  $n \ge 2$ , Central series, Nilpotent groups and their properties, Upper and lower central series.

Composition series, Zassenhaus lemma, Jordan-Holder theorem.

#### Section-III

Modules, Cyclic modules, Simple modules, Schur lemma, Free modules, Torsion modules, Torsion free modules, Fundamental structure theorem for finitely generated free modules, Modules over principal ideal domain and its applications to finitely generated abelian groups.

#### Section-IV

Noetherian and Artinian modules, Noetherian and Artinian rings, Nil and nilpotent ideals in Noetherian and Artinian rings, Hilbert basis theorem.

 $\operatorname{Hom}_{R}(\mathbf{R},\mathbf{R})$ , Opposite rings, Wedderburn-Artin theorem, Maschke theorem.

- 1. I. S. Luther and I.B.S. Passi, Algebra, Vol. I-Groups, Narosa Publishing House, 2013.
- 2. I. S. Luther and I.B.S. Passi, Algebra, Vol. III-Modules, Narosa Publishing House, 2013.
- Charles Lanski, Concepts in Abstract Algebra, American Mathematical Society, First Indian Edition, 2010.
- 4. Vivek Sahai and Vikas Bist, Algebra, Narosa Publishing House, 1999.
- D. S. Malik, J. N. Mordenson and M. K. Sen, Fundamentals of Abstract Algebra, McGraw-Hill International Edition, 1997.
- 6. P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra (2nd Edition), Cambridge University Press, Indian Edition, 1997.
- 7. C. Musili, Introduction to Rings and Modules, Narosa Publication House, 1994.
- 8. N. Jacobson, Basic Algebra, Vol. I and II, W.H Freeman, 1980.
- 9. M. Artin, Algebra, Prentice-Hall of India, 1991.
- 10. Ian D. Macdonald, The Theory of Groups, Clarendon Press, 1968.

# **MAT-102:** Mathematical Analysis

#### Time : 3 hours

Max. Marks : 80 Credits : 4:0:0

Note: The question paper will consist of five Sections. Each of the sections I to IV will contain two questions and the students shall be asked to attempt one question from each. Section-V shall be compulsory and will contain eight short answer type questions without any internal choice covering the entire syllabus.

#### Section-I

Riemann-Stieltjes integral, Existence and properties, Integration and differentiation, The fundamental theorem of calculus, Integration of vector-valued functions, Rectifiable curves.

#### Section-II

Sequence and series of functions, Pointwise and uniform convergence, Cauchy criterion for uniform convergence,  $M_n$ -test for uniform convergence, Weierstrass M-test, Abel's and Dirichlet's tests for uniform convergence, Uniform convergence and continuity, Uniform convergence and Integration, Uniform convergence and differentiation, Weierstrass approximation theorem.

#### Section-III

Power series, uniform convergence and uniqueness theorem, Abel's theorem, Tauber's theorem.

Functions of several variables, Linear Transformations, Euclidean space  $\mathbb{R}^n$ , Derivatives in an open subset of  $\mathbb{R}^n$ , Chain Rule, Partial derivatives, Continuously Differentiable Mapping, Young and Schwarz theorems.

#### Section-IV

Taylor theorem, Higher order differentials, Explicit and implicit functions, Implicit function theorem, Inverse function theorem, Change of variables, Extreme values of explicit functions, Stationary values of implicit functions, Lagrange multipliers method, Jacobian and its properties.

- 1. Walter Rudin, Principles of Mathematical Analysis (3rd edition) McGraw-Hill, International Student Edition, 1976.
- 2. T. M. Apostol, Mathematical Analysis, Narosa Publishing House, New Delhi, 1974.
- 3. H. L. Royden, Real Analysis, Macmillan Pub. Co., Inc. 4th Edition, New York, 1993.
- 4. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH Pub. Co. Pvt. Ltd, 1976.
- 5. R. G. Bartle, The Elements of Real Analysis, Wiley International Edition, 2011.
- S. C. Malik and Savita Arora, Mathematical Analysis, New Age International Limited, New Delhi, 2012.

# **MAT-103:** Ordinary Differential Equations

#### Time : 3 hours

Max. Marks : 80 Credits : 4:0:0

Note: The question paper will consist of five Sections. Each of the sections I to IV will contain two questions and the students shall be asked to attempt one question from each. Section-V shall be compulsory and will contain eight short answer type questions without any internal choice covering the entire syllabus.

#### Section-I

Preliminaries,  $\epsilon$ -approximate solution, Cauchy-Euler construction of an  $\epsilon$ -approximate solution of an initial value problem, Equicontinuous family of functions, Ascoli-Arzela Lemma, Cauchy-Peano existence theorem.

Lipschitz condition, Picard-Lindelof existence and uniqueness theorem for  $\frac{dy}{dt} = f(t, y)$ , Solution of initial-value problems by Picard's method, Dependence of solutions on initial conditions. (Relevant topics from the books by Coddington and Levinson, and Ross).

#### Section-II

Linear systems, Matrix method for homogeneous first order system of linear differential equations, Basic theory of the homogeneous linear system, Fundamental set of solutions, Fundamental matrix of solutions, Wronskian of solutions, Abel-Liouville formula, Non-homogeneous linear system. Strum Theory: Self-adjoint equations of the second order, Some basic results of Sturm theory, Abel's formula, Strum Separation theorem, Strum's Fundamental comparison theorem. (Relevant topics from chapters 7 and 11 of book by Ross)

#### Section-III

Nonlinear differential systems, Phase plane, Path, Critical points, Autonomous systems, Isolated critical point, Path approaching a critical point, Path entering a critical point, Types of critical points - Center, Saddle points, Spiral points, Node points. Stability of critical points, Stable critical points, Asymptotically stable critical points, Unstable critical points, Critical points and paths of linear systems.

(Relevant topics from chapter 13 of book by Ross).

### Section-IV

Almost linear systems, Critical points and paths of almost linear systems, Nonlinear conservative dynamical systems, Dependence on a parameter, Liapunov's direct method.

Limit Cycles and Periodic solutions: Limit cycles, Periodic solutions, Existence and nonexistence of limit cycles, Bendixson's nonexistence criterion, Poincare-Bendixson theorem (statement only), Index of a critical point.

Strum-Liouville problems, Orthogonality of characteristic functions.

(Relevant topics from chapters 12 and 13 of the book by Ross).

- **1.** E. A. Coddington and N. Levinson, Theory of ordinary differential equations, Tata McGraw Hill, 2000.
- 2. S. L. Ross, Differential equations, John Wiley and Sons Inc., New York, 1984.
- **3.** W. E. Boyce and R. C. Diprima, Elementary differential equations and boundary value problems, John Wiley and Sons, Inc., New York, 4th edition, 1986.
- 4. G. F. Simmon, Differential Equations, Tata McGraw Hill, New Delhi, 1993.

# MAT-104: Complex Analysis

#### Time : 3 hours

Max. Marks : 80 Credits : 4:0:0

Note: The question paper will consist of five Sections. Each of the sections I to IV will contain two questions and the students shall be asked to attempt one question from each. Section-V shall be compulsory and will contain eight short answer type questions without any internal choice covering the entire syllabus.

#### Section-I

Functions of a complex variable, Continuity, Differentiability, Analytic functions and their properties, Cauchy-Riemann equations in Cartesian and polar coordinates.

Power series, Radius of convergence, Differentiability of sum function of a power series, Branches of many valued functions with special reference to  $\arg z$ ,  $\log z$  and  $z^a$ .

#### Section-II

Path in a region, Contour, Complex integration, Cauchy theorem, Cauchy integral formula, Extension of Cauchy integral formula for multiple connected domain, Poisson integral formula, Higher order derivatives, Complex integral as a function of its upper limit, Morera theorem, Cauchy inequality, Liouville theorem, Taylor theorem.

#### Section-III

Zeros of an analytic function, Laurent series, Isolated singularities, Cassorati-Weierstrass theorem, Limit point of zeros and poles. Maximum modulus principle, Schwarz lemma, Meromorphic functions, Argument principle, Rouche theorem, Fundamental theorem of algebra, Inverse function theorem.

#### Section-IV

Calculus of residues, Cauchy residue theorem, Evaluation of integrals of the types

$$\int_0^{2\pi} f(\cos\theta, \sin\theta) d\theta, \ \int_{-\infty}^{\infty} f(x) dx, \ \int_0^{\infty} f(x) \sin mx \ dx \ \text{ and } \int_0^{\infty} f(x) \cos mx \ dx.$$

Conformal mappings, Space of analytic functions and their completeness, Hurwitz theorem, Montel theorem, Riemann mapping theorem.

- 1. H. A. Priestly, Introduction to Complex Analysis, Clarendon Press, Oxford, 1990.
- 2. J. B. Conway, Functions of One Complex Variable, Springer-Verlag, International Student Edition, Narosa Publishing House, 2002.
- Ruel V. Churchill and James Ward Brown, Complex Variables and Applications, McGraw-Hill Publishing Company, 2009.
- 4. E. T. Copson, An Introduction to the Theory of Functions of a Complex Variable, Oxford University Press, London, 1972.

- 5. E. C. Titchmarsh, The Theory of Functions, Oxford University Press, London.
- 6. H. S. Kasana, Complex Variables: Theory and Applications, PHI Learning Private Ltd, 2013.
- 7. Dennis G. Zill and P. D. Shanahan, A First Course in Complex Analysis with Applications, John Bartlett Publication, 2nd Edition, 2010.

# **MAT-105:** Mathematical Statistics

#### Time : 3 hours

Max. Marks : 80 Credits : 4:0:0

Note: The question paper will consist of five Sections. Each of the sections I to IV will contain two questions and the students shall be asked to attempt one question from each. Section-V shall be compulsory and will contain eight short answer type questions without any internal choice covering the entire syllabus.

#### Section-I

Probability: Definition and various approaches of probability, Addition theorem, Boole's inequality, Conditional probability and multiplication theorem, Independent events, Mutual and pairwise independence of events, Bayes' theorem and its applications.

#### Section-II

Random variable and probability functions: Definition and properties of random variables, Discrete and continuous random variables, Probability mass and density functions, Distribution function, Concepts of bivariate random variable: joint, marginal and conditional distributions.

Mathematical expectation: Definition and its properties, Variance, Covariance, Moment generating function- Definitions and their properties.

#### Section-III

Discrete distributions: Uniform, Bernoulli, Binomial, Poisson and Geometric distributions with their properties.

Continuous distributions: Uniform, Exponential and Normal distributions with their properties.

### Section-IV

Testing of hypothesis: Parameter and statistic, Sampling distribution and standard error of estimate, Null and alternative hypotheses, Simple and composite hypotheses, Critical region, Level of significance, One tailed and two tailed tests, Two types of errors.

Tests of significance: Large sample tests for single mean, Single proportion, Difference between two means and two proportions.

- A. M. Mood, F. A. Graybill, and D. C. Boes, Introduction to the Theory of Statistics, McGraw-Hill,1974.
- 2. S. C. Gupta and V. K. Kapoor, Fundamentals of Mathematical Statistics, Sultan Chand and Sons, New Delhi.
- 3. J. E. Freund, Mathematical Statistics, Prentice Hall College Div, 1992.
- 4. M. Spiegel, Probability and Statistics, Schaum Outline Series.

# MAT-107: Mathematical Lab-I

Max. Marks : 50 Credits : 0:0:2

Mathematical problem solving techniques based on paper MAT-101 to MAT-105 will be taught. There will be problems based on 5-6 problem solving techniques from each paper.

**Note:** Every student will have to maintain practical record of atleast 25 problems solved during practical class work in a file. Examination will be conducted through a question paper set jointly by the external and internal examiners. The question paper will consists of four questions based on problem solving techniques/algorithm. An examinee will be asked to write the solutions of any two in the answer book. Evaluation will be made on the basis of the examinee's performance in written solutions, practical record and viva-voce.

Practical examination will be conducted as per the following distribution of marks: Writing solutions of problems : 20 marks Viva Voce : 20 marks Practical record : 10 marks.

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# Semester-II

# **Core Courses**

Course	Title of the Course	Theory	Internal	Practical	Credits	Contact	Total
Code		Marks	Marks	Marks	L:T:P	hrs	Credits
						per week	
MAT-201	Field Extensions and	80	20	-	4:0:0	4	4
	Galois Theory						
MAT-202	Measure and	80	20	-	4:0:0	4	4
	Integration Theory						
MAT-203	Integral Equations and	80	20	-	4:0:0	4	4
	Calculus of Variations						
MAT-204	General Topology	80	20	-	4:0:0	4	4
MAT-205	Computing Lab-I	-	-	50	0:0:2	4	2
	(Documentation in LaTex)						
MAT-206	Seminar	-	-	25	-	-	1
MAT-207	Self Study Paper	-	-	25	-	-	1

# Discipline Centric Elective Courses (Any one)

Course	Title of the Course	Theory	Internal	Credits	Contact	Total
Code		Marks	Marks	L:T:P	hrs	Credits
					per week	
MAT-208	Operations Research Techniques	80	20	4:0:0	4	4
MAT-209	Information Theory	80	20	4:0:0	4	4

# Foundation Elective Courses (Any one)

Course	Title of the Course	Theory	Internal	Practical	Credits	Contact	Total
Code		Marks	Marks	Marks	L:T:P	hrs	Credits
						per week	
MAT-210	Value Education	40	10	-	2:0:0	2	2
MAT-211	Communication Skills and	40	10	-	2:0:0	2	2
	Personality Development						

Total Credits : 26 Total Contact Hours per Week : 26 Max Marks : 650

# MAT-201: Field Extensions and Galois Theory

#### Time : 3 hours

Max. Marks : 80 Credits : 4:0:0

Note: The question paper will consist of five Sections. Each of the sections I to IV will contain two questions and the students shall be asked to attempt one question from each. Section-V shall be compulsory and will contain eight short answer type questions without any internal choice covering the entire syllabus.

#### Section-I

Fields, Prime fields, Finite field extensions, Degree of field extensions, Simple Extensions, Algebraic extensions, Splitting fields, Algebraically closed fields.

#### Section-II

Separable and inseparable extensions, Perfect fields.

Monomorphisms and their linear independence, Automorphism of fields, Fixed fields, Normal extensions, The fundamental theorem of Galois theory.

#### Section-III

Finite fields, Existence of  $GF(p^n)$ , Construction of finite fields, Primitive elements, Langrage's theorem on primitive elements, Roots of unity, Cyclotomic polynomials, Cyclotomic extensions of rational number field.

#### Section-IV

Solutions by radicals, Extension by radicals, Generic polynomial, Insolvability of the general polynomial of degree  $n \ge 5$  by radicals, Ruler and compasses construction.

- 1. I. S. Luther and I.B.S.Passi, Algebra, Vol. IV-Field Theory, Narosa Publishing House, 2012.
- 2. Ian Stewart, Galois Theory, Chapman and Hall/CRC, 2004.
- 3. Vivek Sahai and Vikas Bist, Algebra, Narosa Publishing House, 1999.
- 4. P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra (2nd Edition), Cambridge University Press, Indian Edition, 1997.
- 5. S. Lang, Algebra, 3rd edition, Addison-Wesley, 1993.
- 6. Ian T. Adamson, Introduction to Field Theory, Cambridge University Press, 1982.
- 7. I. N. Herstein, Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.

# MAT-202: Measure and Integration Theory

Time : 3 hours

Max. Marks : 80 Credits : 4:0:0

Note: The question paper will consist of five Sections. Each of the sections I to IV will contain two questions and the students shall be asked to attempt one question from each. Section-V shall be compulsory and will contain eight short answer type questions without any internal choice covering the entire syllabus.

#### Section-I

Set functions, Intuitive idea of measure, Elementary properties of measure, Measurable sets and their fundamental properties. Lebesgue measure of sets of real numbers, Algebra of measurable sets, Borel sets and their measurability, Equivalent formulation of measurable sets in terms of open, closed,  $F_{\sigma}$  and  $G_{\delta}$  sets, Non-measurable sets.

#### Section-II

Measurable functions and their equivalent formulations, Properties of measurable functions, Approximation of a measurable function by a sequence of simple functions, Measurable functions as nearly continuous functions, Egoroff's theorem, Lusin's theorem, Convergence in measure and F. Riesz theorem for convergence in measure, Almost uniform convergence.

#### Section-III

Shortcomings of Riemann Integral, Lebesgue Integral of a bounded function over a set of finite measure and its properties, Lebesgue integral as a generalization of Riemann integral, Bounded convergence theorem, Integral of non-negative functions, Fatou's Lemma, Monotone convergence theorem, General Lebesgue Integral, Lebesgue convergence theorem.

#### Section-IV

Vitali's covering lemma, Differentiation of monotonic functions, Functions of bounded variation and their representation as difference of monotonic functions, Differentiation of indefinite integral, Fundamental theorem of calculus, Absolutely continuous functions and their properties, Convex functions, Jensen's Inequality.

- 1. H. L. Royden, Real Analysis, Macmillan Pub. Co., Inc. 4th Edition, New York, 1993.
- 2. P. K. Jain and V. P. Gupta, Lebesgue Measure and Integration, New Age International (P) Limited Published, New Delhi, 1986.
- 3. G. De Barra, Measure Theory and Integration, Wiley Eastern Ltd., 1981.
- 4. Walter Rudin, Principles of Mathematical Analysis (3rd edition) McGraw-Hill, Kogakusha, 1976, International Student Edition.
- 5. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH Pub. Co. Pvt. Ltd, 1976.
- 6. R. G. Bartle, The Elements of Real Analysis, Wiley International Edition, 2011.

# MAT-203: Integral Equations and Calculus of Variations

Time : 3 hours

Max. Marks : 80 Credits : 4:0:0

Note: The question paper will consist of five Sections. Each of the sections I to IV will contain two questions and the students shall be asked to attempt one question from each. Section-V shall be compulsory and will contain eight short answer type questions without any internal choice covering the entire syllabus.

#### Section-I

Linear Integral equations, Some basic identities, Initial value problems reduced to Volterra integral equations, Methods of successive substitution and successive approximation to solve Volterra integral equations of second kind, Iterated kernels and Neumann series for Volterra equations, Resolvent kernel as a series, Laplace transform method for a difference kernel, Solution of a Volterra integral equation of the first kind.

#### Section-II

Boundary value problems reduced to Fredholm integral equations, Methods of successive approximation and successive substitution to solve Fredholm equations of second kind, Iterated kernels and Neumann series for Fredholm equations, Resolvent kernel as a sum of series, Fredholm resolvent kernel as a ratio of two series, Fredholm equations with separable kernels, Approximation of a kernel by a separable kernel, Fredholm Alternative, Non homogeneous Fredholm equations with degenerate kernels.

### Section-III

Green's function, Use of method of variation of parameters to construct the Green's function for a non-homogeneous linear second order boundary value problem, Basic four properties of the Green's function, Alternate procedure for construction of the Green's function by using its basic four properties. Reduction of a boundary value problem to a Fredholm integral equation with kernel as Green's function, Hilbert-Schmidt theory for symmetric kernels.

#### Section-IV

Motivating problems of calculus of variations, Shortest distance, Minimum surface of resolution, Brachistochrone problem, Isoperimetric problem, Geodesics, Fundamental lemma of calculus of variations, Euler's equation for one dependant function and its generalization to 'n' dependant functions and to higher order derivatives, Conditional extremum under geometric constraints and under integral constraints.

- 1. A. J. Jerri, Introduction to Integral Equations with Applications, A Wiley Interscience Publication, 1999.
- 2. R. P. Kanwal, Linear Integral Equations, Theory and Techniques, Academic Press, New York.
- 3. J. M. Gelfand and S.V. Fomin, Calculus of Variations, Prentice Hall, New Jersy, 1963.
- 4. W. V. Lovitt, Linear Integral Equations, McGraw Hill, New York.
- 5. F. B. Hilderbrand, Methods of Applied Mathematics, Dover Publications.

# MAT-204: General Topology

### Time : 3 hours

Max. Marks : 80 Credits : 4:0:0

Note: The question paper will consist of five Sections. Each of the sections I to IV will contain two questions and the students shall be asked to attempt one question from each. Section-V shall be compulsory and will contain eight short answer type questions without any internal choice covering the entire syllabus.

#### Section-I

Definition and examples of topological spaces, Comparison of topologies on a set, Intersection and union of topologies on a set, Limit point of a set, Derived set, Closed set, Closure of a set, Kuratowski closure axioms, Closure operator, Dense sets, Interior point and Interior of a set, Interior axioms, Exterior of a set, Exterior axioms, Boundary of a set, Interior, exterior and boundary operators, Neighborhoods, Alternative methods of defining a topology in terms of neighborhood system and Kuratowski closure operator.

#### Section-II

Relative (Induced) topology, Base and subbase for a topology, Base for neighbourhood system.

Continuous functions, Composition of continuous functions, Pasting lemma, Open and closed functions, Homeomorphisms, Topological properties.

Connectedness and its characterization, Connected subsets and their properties, Continuity and connectedness, Components, Locally connected spaces.

#### Section-III

Separation axioms:  $T_0$ ,  $T_1$ ,  $T_2$ -spaces, their characterization and basic properties,  $T_2$ -spaces and sequences.

First countable, Second countable and Separable spaces, Hereditary and topological property, Countability of a collection of disjoint open sets in separable and second countable spaces, Lindelöf theorem.

#### Section-IV

Compact spaces and subsets, Compactness in terms of finite intersection property, Continuity and compact sets, Basic properties of compactness, Closedness of compact subset of a Hausdorff space and of a continuous map from a compact space into a Hausdorff and its consequence. Sequentially and Countably compact spaces, Locally compact spaces and One point compactification.

- 1. W. J. Pervin, Foundations of General Topology, Academic Press Inc. New York, 1964.
- 2. C. W. Patty, Foundation of Topology, Jones and Bartlett, 2009.
- 3. Fred H. Croom, Principles of Topology, Cengage Learning, 2009.
- 4. K. D. Joshi, Introduction to General Topology, New Age International, 1983.
- 5. J. L. Kelly, General Topology, Springer Verlag, New York, 2000.
- 6. J. R. Munkres, Topology, Pearson Education, 2002.
- 7. K. Chandrasekhara Rao, Topology, Alpha Science International, 2009.

# MAT-205: Computing Lab-I (Documentation in LaTex)

### Time : 4 hours

### Max. Marks : 50 Credits : 0:0:2

- Note: (A) Each candidate will be provided a question paper of four questions and will be required to attempt two questions. The candidate will first prepare the docoment in LaTex of the questions in the answer-book and then run the same on the computer, and then finally add the print-outs of these programs in the answer-book. This work will consist of 30 marks, 15 marks for each question.
  - (B) The practical file of each student will be checked and Viva-Voce examination based upon the practical file and the theory will be jointly conducted by external and internal examiners. This part of the practical examination shall be of 20 marks.

### List of Practicals

1. Create a document in LATEX to output the following:

**Hello World!** Today I am learning  $IAT_EX$ .  $IAT_EX$  is a great program for writing math. I can write in line math such as  $a^2 + b^2 = c^2$ . I can also write equation in a new line:

$$\gamma^2 + \theta^2 = \omega^2$$

2. Write a program in LAT<sub>F</sub>X to output the following:

$(a_{11})$	$a_{12}$		$a_{1n}$	$v_1$	$w_1$
$a_{21}$	$a_{22}$		$a_{2n}$	$v_2$	$w_2$
÷	÷	·	:	:	=
$a_{n1}$	$a_{n2}$		$a_{nn}$	$v_n$	$w_n$

3. Write a program in LATEX to obtain the following output:

$$a_1x + b_1y + c_1z = d_1$$
  

$$a_2x + b_2y + c_2z = d_2$$
  

$$a_3x + b_3y + c_3z = d_3$$

4. Write a program in LATEX to obtain the following output: Find the solution of the Laplace Equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

in the rectangle  $0 \le x \le a, 0 \le y \le b$  satisfying the boundary conditions

$$u(x, 0) = 0, u(x, b) = 0$$
  
 $u(0, y) = 0, u(a, y) = f(y)$ 

5. Write a program in  ${\rm IAT}_{\rm E}{\rm X}$  to create the following table:

Table 1: This is a table								
x	1	2	3					
f(x)	4	8	12					
f(x)	4	8	12					

- 6. Write a program in  ${\rm IAT}_{\rm E}{\rm X}$  with the following output:

$$\int_{a}^{b} x \, dx = \left. \frac{x^2}{2} \right|_{a}^{b}$$
$$\iiint_{V} f(x, y, z) \, dV = F$$

7. Write a program in  ${\rm IAT}_{\rm E} {\rm X}$  with the following output:

$$\frac{df}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$|x| = \begin{cases} -x, & \text{if } x < 0\\ x, & \text{if } x \ge 0 \end{cases}$$

8. Write a program in  ${\rm IAT}_{\rm E} {\rm X}$  with the following output:

$$F(x) = A_0 + \sum_{n=1}^{N} \left[ A_n \cos\left(\frac{2\pi nx}{P}\right) + B_n \sin\left(\frac{2\pi nx}{P}\right) \right]$$
$$\sum_n \frac{1}{n^s} = \prod_p \frac{1}{1 - \frac{1}{p^s}}$$

9. Write a program in  $\ensuremath{\mathbb{I}}\xspace{-1.5}\ensuremath{\mathbb{I}}\xspace{-1.5}\xspace{-1.5}\ensuremath{\mathbb{I}}\xspace{-1.5$ 

$$f(x) = x^{2} + 3x + 5x^{2} + 8 + 6x$$
  
=  $6x^{2} + 9x + 8$   
=  $x(6x + 9) + 8$ 

10. Write a program in  ${\rm L\!A}\!T_{\rm E}\!X$  with the following output:

$$X = \frac{F_0}{k} \frac{1}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}}$$
$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

11. Write a program in LATEX with the following output:

$$\begin{split} & 6\mathrm{CO}_2 + 6\mathrm{H}_2\mathrm{O} \to \mathrm{C}_6\mathrm{H}_{12}\mathrm{O}_6 + 6\mathrm{O}_2 \\ & \mathrm{SO}_4^{2-} + \mathrm{Ba}^{2+} \to \mathrm{Ba}\mathrm{SO}_4 \end{split}$$

12. Write a program in  ${\rm IAT}_{\rm E} \! X$  with the following output:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} - \nu \nabla^2 (\mathbf{u}) = -\nabla \mathbf{h}$$
$$m\ddot{x} + c\dot{x} + kx = F_0 \sin(2\pi f t)$$
$$\alpha A\beta B\gamma \Gamma \delta \Delta \pi \Pi \omega \Omega$$

13. Write a program in LATEX with the following output:

"Maxwell's equations" are named for James Clark Maxwell and are as follows:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \text{Gauss's Law} \qquad (0.0.1)$$
  
$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \text{Gauss's Law for Magnetism} \qquad (0.0.2)$$
  
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \text{Faraday's Law of Induction} \qquad (0.0.3)$$
  
$$\vec{\nabla} \times \vec{B} = \mu_0 \left(\epsilon_0 \frac{\partial \vec{E}}{\partial t} + \vec{J}\right) \qquad \text{Ampere's Circuital Law} \qquad (0.0.4)$$

Equations (0.0.1), (0.0.2), (0.0.3), and (0.0.4) are some of the most important equations in Physics.

14. Write a program in  ${\rm IAT}_{\rm E}\!{\rm X}$  with the following output:

$$\frac{8}{-40}$$

15. Write a program in  ${\rm I\!AT}_{\rm E}\!{\rm X}$  with the following output:

Let f and g be bounded measurable functions defined on a set E of finite measure. Then

$$\int_E f + g = \int_E f + \int_E g.$$

If  $f_n \xrightarrow{m} f$  and  $g_n \xrightarrow{m} g$ , then show that  $f_n + g_n \xrightarrow{m} f + g$ .

16. Write a program in  ${\rm IAT}_{\rm E\!X}$  to display your own photograph.

# MAT-208: Operations Research Techniques

Time : 3 hours

Max. Marks : 80 Credits : 4:0:0

Note: The question paper will consist of five Sections. Each of the sections I to IV will contain two questions and the students shall be asked to attempt one question from each. Section-V shall be compulsory and will contain eight short answer type questions without any internal choice covering the entire syllabus.

#### Section-I

**Operations Research:** Origin, Definition and scope. **Linear Programming:** Formulation and solution of linear programming problems by graphical and simplex methods, Big-M and two-phase methods, Degeneracy, Duality in linear programming.

#### Section-II

**Transportation Problems:** Basic feasible solutions, Optimum solution by stepping stone and modified distribution methods, Unbalanced and degenerate problems, Transshipment problem. **Assignment problems:** Solution by Hungarian method, Unbalanced problem, Case of maximization, Travelling salesman and crew assignment problems.

#### Section-III

**Queuing models:** Basic components of a queuing system, Concepts of stochastic processes, Poisson process, Birth-death process. Steady-state solution of Markovian queuing models with single and multiple servers (M/M/1, M/M/C, M/M/1/k, M/MC/k).

**Sequencing problems:** Solution of sequecing problems, processing n jobs through 2 machines, n jobs through 3 machines, n jobs through m machines, 2 jobs through m machines.

#### Section-IV

**Inventory control models:** Economic order quantity(EOQ) model with uniform demand and with different rates of demands in different cycles, EOQ when shortages are allowed, EOQ with uniform replenishment, Inventory control with price breaks.

**Game Theory:** Two person zero sum game, Game with saddle points, The rule of dominance, Algebraic, Graphical and linear programming methods for solving mixed strategy games.

- 1. H. A. Taha, Operations Research-An Introduction, Prentice Hall, 1997.
- 2. P. K. Gupta and D.S. Hira, Operations Research, S. Chand and Co. Ltd., 2014.
- 3. S. D. Sharma, Operations Research, Kedar Nath Ram Nath Publications.
- 4. J. K. Sharma, Mathematical Models in Operations Research, Tata McGraw-Hill Publishing Company Ltd., 1989.
- Kanti Swarup, P. K. Gupta and ManMohan, Operations Research, Sultan Chand and Sons, New Delhi, 2005.

# MAT-209: Information Theory

### Time : 3 hours

Max. Marks : 80 Credits : 4:0:0

Note: The question paper will consist of five Sections. Each of the sections I to IV will contain two questions and the students shall be asked to attempt one question from each. Section-V shall be compulsory and will contain eight short answer type questions without any internal choice covering the entire syllabus.

#### Section-I

Measure of information Axioms for a measure of uncertainty. The Shannon entropy and its properties. Joint and conditional entropies. Trans-information and its properties. Axiomatic characterization of the Shannon entropy due to Shannon and Fadeev.

#### Section-II

Noiseless coding - Ingredients of noiseless coding problem. Uniquely decipherable codes. Necessary and sufficient condition for the existence of instantaneous codes. Construction of optimal codes.

#### Section-III

Discrete memoryless channel - Classification of channels. Information processed by a channel. Calculation of channel capacity. Decoding schemes. The ideal observer. The fundamental theorem of information theory.

#### Section-IV

Continuous channels - The time-discrete Gaussian channel. Uncertainty of an absolutely continuous random variable. The converse to the coding theorem for time-discrete Gaussian channel. The time-continuous Gaussian channel. Band-limited channels.

- 1. R. Ash, Information Theory, Interscience Publishers, New York, 1965.
- 2. F.M. Reza, An Introduction to Information Theory, MacGraw-Hill Book Company Inc., 1961.
- **3.** J. Aczela dn Z. Daroczy, On Measures of Information and their Characterizations, Academic Press, New York.